1404-01

No. of Pages : 8

Degree (Part-I) Vocational Examination, 2019

MATHEMATICS

| First Paper |

| PPU-DI(V)-(SUB)-MATH |

Time : Three Hours|

|Maximum Marks :100

Note : Candidates are required to give their answers in their own words as far as practicable. The questions are of equal value. Answer **any five** questions, selecting at least one from each group.

GROUP-A

1. (a) Define equivalence relations. Prove that the relation R on Z defined by "aRb if 3 divides a-b" is an equivalence relation. Note that Z denotes the set of all integers.

(b) Prove that the function $f : R^+ \to R$ defined by f(x) = log x is one-one and onto. Here R^+ is the set of all positive real numbers and R is the set of all real numbers.

2. (a) Let A = {a, b, c}, B = {1, 2, 3} and C = {x, y, z}. Find A × B, A × C and $A \times (B \cup C)$. Is $A \times (B \cup C) = (A \times B) \cup (A \times C)$?

(b) Prove that composite of two relations on a given set X is also a relation on X.

3. (a) Prove that set G of all cube roots of unity forms a cyclic group w.r.t multiplication of complex numbers.

(b) Prove that the permutation group P_3 = {I, (12), (13), (23), (123), (132)} is not abelian. Also find the order of element (1,2,3).

4. (a) Define nngs. Give an example of ring without unity.

(b) Let R be a ring and $b, a \in R$. Prove the following:

(i) a.0 = 0

(ii) a (-b) = -(ab) = (-a).b

GROUP-B

5. (a) Define Hermitian and Skew-Hermitian matrices.

Let $A = \begin{pmatrix} 1+i & 2 \\ -3 & 1-5i \end{pmatrix}$. Determine a Hermitian matrix P and a Skew-Hermitian matrix Q such that A = P + Q.

(b) Define unitary matrices. Let

$$A = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1+i \\ 1-i & -1 \end{pmatrix}.$$

Prove that A is a unitary matrix.

6. (a) Find the rank of matrix :

$$\begin{pmatrix} 1 & 1 & 1 & -1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

(b) Find A^{-1}, where $A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 4 \end{pmatrix}$.

7. (a) Prove that the set $S = \{(x, y, z) \in R^3 \mid x + y + z \leq 1\}$ is a convex set.

(b) Solve graphically the following L P P :

Min. $Z = x_1 + x_2$

subject to constraints :

$$x_1 + x_2 \geq 5$$

$$x_1 + x_2 \leq 3$$

$$x_2 \leq 6$$

and $x_1, x_2 \geq 0$.

8 Using Simplex method, solve the following L.P.P. :

Maximize $Z = 3x_1 + 2x_2$

subject to constraints :

$$x_1 + x_2 \leq 4$$

$$x_1 - x_2 \leq 2$$

and $x_1, x_2 \geq 0$

GROUP-C

9 (a) If $2\cos\theta = x + \frac{1}{x}$ and $2\cos\phi = y + \frac{1}{y}$,

prove that the values of

$$x'''y'' + \frac{1}{x'''y''} = 2\cos(m\theta + n\phi)$$

(b) If $\sin\alpha + \sin\beta + \sin\gamma = 0 = \cos\alpha + \cos\beta + \cos\gamma$, then prove that $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$

10. (a) If $\sin(\alpha + i\beta) = x + iy$, then prove that :

$$\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1$$

(b) Find the sum of series :

$$\left(\frac{2}{3} + \frac{1}{7}\right) - \frac{1}{3}\left(\frac{2}{3^3} + \frac{1}{7^3}\right) + \frac{1}{5}\left(\frac{2}{3^5} + \frac{1}{7^5}\right) \text{.....adinf}$$

11. (a) Test the convergence of the series :

$$\left(\frac{2^2}{1^2} - \frac{2}{1}\right)^{-1} + \left(\frac{3^3}{2^3} - \frac{3}{2}\right)^{-2} + \left(\frac{4^4}{3^4} - \frac{4}{3}\right)^{-3} + \text{.....adinf}$$

(b) Define alternating series. Prove that the series :

$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \text{.....adinf}$ is convergent but not absolutely convergent.

12. (a) Prove that every convergent sequence i bounded

(b) Let $\sum u_n$ be a convergent sequence. Prove tha $\lim_{n\to\infty} u_n = 0$. Also give an example of a series $\sum u_n$ for which $\lim_{n\to\infty} u_n = 0$, but the series is not convergent.

13. (a) Prove that the function f defined as :

$$f(x) = \begin{cases} x & , \quad x \text{ is rational} \\ 1 - x & , \quad x \text{ is irrational} \end{cases}$$

is continuous only at x = 1/2.

(b) Define continuity of a function. Give an example of a function on R which is discontinuous at x = 0.

GROUP-D

14. (a) Find the condition that a straight line y = mx +c may touch the circle $x^2 + y^2 = a^2$.

(b) Find the equation to the circle which passes through the points (1, 0), (0, -6) and (3, 4).

15 (a) Reduce the conic $12x^2 - 23xy + 10y^2 - 25x + 26y - 14 = 0$ to its standard form.

(b) Find the vertex, axis, focus and latus rectum of the conic $4y^2 + 12x - 20y + 67 = 0$

GROUP-E

16. (a) Define direction angles and direction cosines. Find direction cosines of a line joining points (1, 0, -1) and (2, 0, 1).

(b) Find the equation of the plane through the points (2, 2, 1) and ((9, 3, 6) and is perpendicular to the plane $2x + 6y + 6z = 9$.

17. (a) Find the equation of the following straight line in symmetrical form :

$$\left.\begin{aligned} x + y + z &= 1 \\ x - y + 2z &= 2 \end{aligned}\right\}$$

(b) For what values of c the lines

$$\frac{x-1}{-3} = \frac{y-2}{2c} = \frac{z-3}{2} \text{ and}$$

$$\frac{x-1}{3c} = \frac{y-5}{1} = \frac{z-6}{-5}$$

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