

2008

TIME : 3 Hours

Full Mark : 100

Candidate are required to give their answers in their own words as far as practicable.

"Answer any eight questions select at least two from each group. The questions are of equal value."

Group-A

1. (a) State and prove Taylor's theorem.
(b) If $y = x^2 \sin x$ find y_n .
2. (a) State and prove Euler's theorem on homogeneous functions of two variable.
(b) If $u = \tan^{-1} \frac{y}{x}$ prove that $\frac{\delta^2 u}{\delta x^2} + \frac{\delta^2 u}{\delta y^2} = 0$.
3. (a) Evaluate $\lim_{x \rightarrow 0} \frac{(1+x)^{1/x} - e}{x}$.
(b) Find the radius of curvature for the cartesian curve $y = f(x)$.

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4. (a) Obtain a reduction formula for $\int \cos^n x dx$.
(b) Evaluate $\int_0^1 \sin x dx$ as the limit of a sum.
5. (a) Find the area of the loop of the curve $y^2 = x(x-1)^2$.
(b) Find the perimeter of the loop of the curve $9ay^2 = (x-2a)(x-5a)^2$.
6. Define Beta and Gamma function, and Prove that $B(m, n) = \frac{\Gamma(m) \Gamma(n)}{\Gamma(m+n)}$.
7. Solve any two of the following :
(a) $y dx = x dy = xy dx$
(b) $\frac{dy}{dx} = \frac{y}{x} + \tan \frac{y}{x}$
(c) $(x-y)^2 \frac{dy}{dx} = a^2$
8. Solve any two of the following :
(a) $x \frac{dy}{dx} + \frac{y^2}{x} = y$
(b) $x dy - y dx - \sqrt{x^2 + y^2} dx = 0$
(c) $\frac{dy}{dx} + 1 = e^{x-y}$

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9. (a) Solve $y = px + p - p^2$ and obtain the singular solution.
(b) Prove that the orthogonal trajectories of the rectangular hyperbola $xy = a^2$ is $x^2 - y^2 = c^2$.

Group - B

10. (a) Establish the formula :
$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$$

(b) Prove that
$$\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = 0$$
11. (a) If $\vec{u}(t)$ and $\vec{v}(t)$ be two differentiable function of the scalar t then show that :
$$\frac{d}{dt} (\vec{u} \cdot \vec{v}) = \vec{u} \cdot \frac{d\vec{v}}{dt} + \frac{d\vec{u}}{dt} \cdot \vec{v}$$

(b) Prove that the necessary and sufficient condition for the vector function $\vec{u}(t)$ to have constant magnitude is $\vec{u} \cdot \frac{d\vec{u}}{dt} = 0$.
12. (a) Prove that $\text{curl} (\vec{u} + \vec{v}) = \text{curl} \vec{u} + \text{curl} \vec{v}$
(b) Prove that $\text{div} (\text{curl} \vec{v}) = 0$.

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13. (a) Find the necessary and sufficient condition for a system of coplaner forces to be in equilibrium.
- (b) Forces P, Q, R act along the line $x=0, y=0$ and $x \cos \alpha + y \sin \alpha = p$. Find the magnitude of the resultant and equation of the line of action.
14. (a) State and prove the principle of virtual work for a coplaner force system.
- (b) Find the work done by the tension of string in a small displacement.
15. write short note on any *two* of the following:-
- (a) Principle of angular momentum.
- (b) Potential energy.
- (c) Inertial frames of reference.
16. (a) A body moves from a point O , so that its acceleration is $\frac{1}{(t+1)^2}$, where t is the time taken in second from O . Find the distance moved in 9 second and the velocity then.
- (b) Given that the accelerations of a point is $(a - bv^2)$ and that when $t=0, s=0$ and $v=0$. Prove that the distance of the point at time t is $\frac{1}{b} \log \cosh(\sqrt{abt})$.

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