16. (a) Discuss SHM and hence find the period of it.

(b) A particle is projected with a velocity v directed away from a point at a distance b from the point of projection. If the acceleration is $\mu \times$ (distance from the fixed point) and always towards the fixed point, find the amplitude of SHM.

17. State and prove principle of conservation of linear momentum, angular momentum and energy.

where $a_0 + A_1 + A_2$ — — are to be determined
in alone not containing h. and
are to be ~~found~~.
D.W. both sides of with respect to h
$f'(x+h) = A_1 + A_2 \cdot 2h + A_3 3h^2 + - - - A_n \cdot nh^{n-1}$ — — (2)
D.B.S with respect to h
$f''(x+h) = A_2 \cdot 2 + A_3 \cdot 6h + - - A_n n(n-1)h^{n-2}$ — — (3)
D.B.S with respect to h
$f'''(x+h) = A_3 \cdot 6 + - - A_n \cdot n(n-1)(n-2)h^{n-3}$ — — (4)
Now putting h = 0 in (1) together

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2011

Time : 3 Hours

Full Marks : 100

The questions are of equal value.

Answer six questions, selecting three from Group-A, one from Group-B and two from Group-C

Group-A

1. (a) State and prove Euler's theorem for a homogeneous function of two variables.

(b) If $y = e^{\tan^{-1}x}$, then prove that

(i) $(1 + x^2)y_2 + (2x - 1)y_1 = 0$

(ii) $(1 + x^2)y_{n+2} + (2nx + 2x - 1)\, y_{n+1} + n(n + 1)y_n = 0$

2. (a) State and prove Taylor's theorem for the expansion of a function in the neighbourhood of a point.

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(b) Evaluate $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}}$

3. (a) Prove that the sum of intercepts of the tangent to $\sqrt{x}+\sqrt{y}=\sqrt{a}$ upon the coordinate axes is constant.

(b) Find the asymptotes to the curve $x^3 + 3x^2y - 4y^2 - x + y + 3 = 0$

4. (a) Find the reduction formula for $\int_0^{\frac{\pi}{2}} \cos^n x\, dx$, where n is an integer.

(b) Evaluate $\int_0^{\frac{\pi}{2}} \cos x\, dx$ from the first principle.

5. (a) Find the area enclosed by the parabolas $y^2 = 4ax$ and $x^2 = 4by$.

(b) Find the length of arc of the curve

$$x^{2/3} + y^{2/3} = a^{2/3}$$

6. Find the volume and surface of the solid generated by revolving the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$ about its base.

7. (a) Solve $\frac{dy}{dx}=\frac{2x+3y+4}{4x+6y+5}$

(b) Solve $\frac{dy}{dx}=\frac{2x-3y}{3x-2y}$

8. (a) Solve $(y - px)(p - 1) = p$

(b) Solve $(1 + y^2)dx = (\tan^{-1} y - x)dy$

9. (a) Solve $(D^2 - a^2)y = \sin ax$

(b) Solve $(D^3 - 2D^2 + D)y = \sin x + e^x$

Group-B

10. (a) Prove that the necessary and sufficient condition for the vector function $\bar{a}(t)$ to have a constant direction is $\bar{a} \times \frac{d\bar{a}}{dt} = 0$.

(b) Prove that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c}$ where $(\vec{c} \times \vec{a}) \times \vec{b} = 0$.

11. (a) Prove that the necessary and sufficient condition for a vector point $\bar{v}$ having constant magnitude is that $\bar{v}\cdot\frac{d\bar{v}}{dt} = 0$.

(b) If $\vec{r} = \vec{a}\cos\omega t + \vec{b}\sin\omega t$, prove that

$$\vec{r} \times \frac{d\vec{r}}{dt} = \omega(\vec{a} \times \vec{b})$$

12. (a) Find the divergence and curl of the vector point function $\vec{F} = (x^2 - y^2)\hat{i} + 2xy\hat{j} + (y^2 - xy)\hat{k}$.

(b) Evaluate (i) $\nabla.(\nabla \times \vec{u})$ and (ii) $\nabla \times (\nabla \times \vec{u})$

13. (a) Obtain the equation of the line of action of the resultant of a system of coplanar forces.

(b) A uniform beam of length 2a rests in equilibrium with one end resting against a smooth vertical wall and with a point of its length resting upon a smooth horizontal rod which is parallel to the wall and at a distance b from it. Show that inclination of the beam to the vertical is $\sin^{-1}\left(\frac{b}{a}\right)^{\frac{1}{3}}$.

Group-C

14. (a) Enumerate the nature of the forces which may be omitted in forming the equation of virtual work.

(b) The middle points of the opposite sides of a quadrilateral are connected by light rods of lengths l and l'. If T and T′ be the tensions in these rods, prove that

$$\frac{T}{l} + \frac{T'}{l'} = 0$$

15. (a) Find the tangential and normal velocity and acceleration of a particle moving a plane curve.

(b) A particle describes the equiangular spiral $r = ae^{m\theta}$ with a constant velocity. Find the components of the velocity and the acceleration along the radius and perpendicular to it.