

2012

Time : 3 hours

Full Marks : 100

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

*Answer any **eight** questions, selecting at least **two** from each group.*

Group - A

1. (a) Evaluate :

$$\lim_{x \to 0}\left(\frac{1}{x} - \cot x\right)$$

(b) If $u = \frac{x-y}{x+y}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

2. (a) State and prove Maclaurin's theorem.

(b) If $y = e^{a \sin^{-1} x}$, prove that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$$

3. (a) Prove that

$$\rho = r\frac{dr}{dp}$$

where the symbols have their usual meanings.

(b) Prove that the sum of the intercepts of the tangents to $\sqrt{x} + \sqrt{y} = \sqrt{a}$ upon the coordinate axes is constant.

4. (a) Obtain the reduction formula for $\int \tan^n x\, dx$.

(b) Find the entire length of the cardioid $r = a(1+\cos\theta)$.

5. (a) Evaluate :

$$\int_a^b \cos x\, dx$$

as a limit of a sum.

(b) Evaluate :

$$\int_0^{\pi/2} \log \sin x\, dx$$

6. Solve any two of the following :

(a) $\dfrac{dy}{dx} = \dfrac{(1+x)y^2}{x^2(1+y)}$

(b) $x\dfrac{dy}{dx} = y + x\tan\dfrac{y}{x}$

(c) $\dfrac{dy}{dx} + 2xy = 2x^3$

7. Define Beta and Gamma function. Prove that

$$\frac{\Gamma(n)\Gamma(m)}{\Gamma(m+n)} = 2\int_0^{\pi/2} (\sin\theta)^{2m-1}(\cos\theta)^{2n-1} d\theta$$

8. Solve any two of the following :

(a) $(x-y-2)\, dx = (2x-2y-3)\, dy$

(b) $(x+1)\dfrac{dy}{dx} - y = e^{3x}(x+1)^2$

(c) $(3x-2y)dy = (2x-3y)dx$

9. (a) Solve $y = 2px + 4xp^2$ and obtain singular solution, where $p = \frac{dy}{dx}$.

(b) Find the orthogonal trajectories of the family of parabolas $y^2 = 4ax$.

Group - B

10. (a) Prove that

$$\left[\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}\right] = 2\left[\vec{a}\,\vec{b}\,\vec{c}\right]$$

(b) Find the volume of parallelopiped whose concurrent edges are expressed by the vectors

$\vec{i} + 2\vec{j} + 3\vec{k}$, $3\vec{i} + 7\vec{j} - 4\vec{k}$ and $\vec{i} - 5\vec{j} + 3\vec{k}$.

11. (a) If $\vec{a}$ and $\vec{b}$ are differentiable vector functions of a scalar t, then show that

$$\frac{d}{dt}\left(\vec{a}\times\vec{b}\right) = \vec{a}\times\frac{d\vec{b}}{dt} + \frac{d\vec{a}}{dt}\times\vec{b}$$

(b) Prorve that the necessary and sufficient condition for the vector function $\vec{a}(t)$ to have constant direction is $\vec{a}\times\frac{d\vec{a}}{dt} = 0$.

12. (a) Prove that

grad $(\phi\psi) = \phi$ grad $\psi + \psi$ grad ϕ

(b) Prove that

curl (grad) $\phi = 0$

Group - C

13. (a) Obtain the equation to the line of action of the resultant of a system of coplanar forces.

(b) Three forces P, Q, R act along the sides of a triangle formed by the lines $x + y = 1$; $y - x = 1$ and $y = 2$. Find the equation to line of action of the resultant.

14. (a) State and prove tha converse of the principle of virtual work for any system of the force in one plane.

(b) Three forces *P*, *Q*, *R*, act along the sides *BC*, *AC*, *BA* of an equilateral triangle *ABC*. If their resultant is a force parallel to *BC* through centroid of the triangle, then prove that $\frac{1}{2}P = Q = R$.

15. (a) State the prove Hooke's law.

(b) State and prove the principle of conservation of linear momentum.

16. (a) When a particle moves under the action of a conservative system of forces the sum of KE and PE is constant throughout the motion. Prove it.

(b) If a particle moving in a plane curve, then find component of its acceleration along the tangent and the normal to the curve at any instant.

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