 XEV (S-II) – M (2)

2nd Yr

2013

Time : 3 hours

Full Marks : 100

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

Answer any six questions, selecting three from group A, one from group B and two from group C.

Group – A

1. (a) State and prove Euler's theorem for a homogeneous function of two variables. — 10

   (b) If $y = e^{ax}\sin(bx+c)$, find y_n. 159

2. (a) State and prove Taylor's theorem for the expansion of a function in the neighbourhood of a point. — 9

   (b) Evaluate $\underset{x\to 0}{Lt}(\cos mx)^{\operatorname{cosec}^2 nx}$

3. (a) Prove that the sum of intercepts of the tangent to $\sqrt{x}+\sqrt{y}=\sqrt{a}$ upon the coordinate axes is constant. — 23

   (b) Find the asymptotes to the curve

   $$y^3 - xy^2 - x^2y + x^3 + x^2 - y^2 = 1$$

4. Find the volume and surface of the solid generated by revolving the circloid $x = a(\theta+\sin\theta)$, $y = a(1+\cos\theta)$ about its base. 55

length

52 5. (a) Find the area of the cardioid $r = a(1+\cos\theta)$.

54 (b) Find the length of the arc of the curve. $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.

38 6. (a) Find the reduction formula for $\int_0^{\frac{\pi}{2}} \cos^n x\,dx$, where n is an integer.

45 (b) Evaluate $\int_0^{\frac{\pi}{2}} \cos x\,dx$ from the first principle.

73 7. (a) Solve $\frac{dy}{dx} = \frac{2x+3y+4}{4x+6y+5}$

(b) Solve $xdx + ydy = \frac{a^2(xdy - ydx)}{(x^2+y^2)}$

75 8. (a) Solve $(y - px)(p-1) = p$

76 (b) Solve $(1+y^2)dx = (\tan^{-1} y - x)dy$

9. (a) Solve $(D^2 - a^2)y = \sec ax$

(b) Solve $(D^3 - 2D^2 + D)y = \sin x + e^x$

Group – B

10. (a) The necessary and sufficient condition for three vectors $\vec{a}, \vec{b}, \vec{c}$ to be coplanar is that the volume of the tetrahedron formed by three vector $\vec{a}, \vec{b}, \vec{c}$ is.

(b) If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors, then

$$\left(\bar{a}\times\bar{b}, \bar{b}\times\bar{c}, \bar{c}\times\bar{a}\right)=\left[\bar{a}\bar{b}\bar{c}\right]^2$$ 126

hence or otherwise show that $\bar{a}\times\bar{b}, \bar{b}\times\bar{c}, \bar{c}\times\bar{a}$ are also non-coplanar.

11. (a) The necessary and sufficient condition for a vector point function $\bar{V}$ having constant magnitude is that $\bar{V}.\frac{d\bar{V}}{dt}=0$. 121

(b) Find the gradient and unit harwel to the level surface $x^2+y-z=1$ at $(1,0,0)$

12. (a) Find the divergence and curl of the vector point fucntion $\bar{F}=\left(x^2-y^2\right)\hat{i}+2xy\hat{j}+\left(y^2-xy\right)\hat{k}$ 123

(b) Prove that $div(\bar{u}\times\bar{v})=\bar{v}.curl\,\bar{u}-\bar{u}.curl\,\bar{v}$

Group – C

13. (a) Find the equation of the line of action of the resultant of a system of coplanar forces acting upon a rigid load.

(b) A uniform beam of length $2a$ rests in equilibrium with one end resting against a smooth vertical wall and with a point of its length resting upon a smooth horizontal rod which is parallel to the wall and at a distance b from it. Show that inclination of the beam to the vertical is $\sin^{-1}\left(\frac{b}{a}\right)^{\frac{1}{3}}$.

14. (a) Discuss the forces which can be omitted in forming the equation of the virtual work.

(b) A strip of length a forms the shorter diagonal of a rhombus of four uniform rods, each of length b and weight w which are hinged together. If one of the rods be supported on a horizontal position, prove that tension of the strip is $\frac{2w(2b^2 - a^2)}{b\sqrt{4b^2 - a^2}}$

15. (a) Find the tangential and normal velocity and acceleration of a particle working a plane curve.

(b) A particle describe the equi-angular spiral $r = ae^{m\theta}$ with a constant velocity. Find the components of the velocity and the acceleration along the radius vector and perpendicular to it.

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16. (a) Discuss SHM and hence find period of it.

(b) A particle starts with a velocity V and works under a retardation equal to K times the space described. Prove that the space traversed before it comes to rest is equal to $\frac{V}{\sqrt{K}}$.

17. State and prove principle of conservation of linear momentum, angular momentum and energy.

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