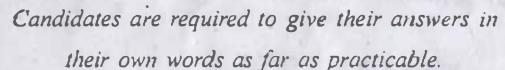
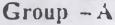
2013

Time: 3 hours Full Marks: 100



The questions are of equal value.

Answer any six questions, selecting three from group A, one from group B and two from group C.



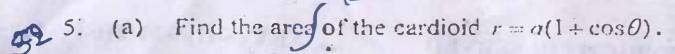
(a) State and prove Euler's theorem for a — (b) homogeneous function of two variables.

(b) If $y = e^{ax} \sin(bx + c)$, find y_a . 159 (620)

- 2. (a) State and prove Taylor's theorem for the expansion of a function in the neighbourhood of a point.
 - (b) Evaluate $L_1(\cos mx)^{\cos ec^2 mx}$
- 3. (a) Prove that the sum of intercepts of the tangent
 - to $\sqrt{x} + \sqrt{y} = \sqrt{a}$ upon the coordinate axes is constant.
 - (b) Find the asymptotes to the curve

$$y^3 - xy^2 - x^2y + x^3 + x^2 - y^2 = 1$$

4. Find the volume and surface of the solid generated by revolving the circloid $x = a(\theta + \sin \theta)$. $y = a(1 + \cos \theta)$ about its base.



(b) Find the length of the arc of the curve,
$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$$
.

6. (a) Find the reduction formula for
$$\int_{0}^{2} \cos^{n} x dx$$
, where n is an integer.

(b) Evaluate
$$\int_{0}^{\frac{\pi}{2}} \cos x \, dx$$
 from the first principle.

7. (a) Solve
$$\frac{dy}{dx} = \frac{2x + 3y + 4}{4x + 6y + 5}$$

(b) Solve
$$xdx + ydy = \frac{a^2(xdy - ydx)}{(x^2 + y^2)}$$

8 (a) Solve
$$(y - px)(p-1) = p$$

Solve
$$(1+y^2)dx = (\tan^{-1}y - x)dy$$

9. (a) Solve
$$(D^2 - a^2)y = \sec ax$$

(b) Solve
$$(D^3 - 2D^2 + D)y = \sin x + e^x$$

Group - B

10. (a) The necessary and sufficient condition for three vectors $\vec{a}, \vec{b}, \vec{c}$ to be coplanar is that the volume of the tetrahedron formed by three vector $\vec{a}, \vec{b}, \vec{c}$ is.

If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors, then

$$(\bar{a} \times \bar{b}, \bar{b} \times \bar{c}, \bar{c} \times \bar{a}) = [\bar{a}\bar{b}\bar{c}]^2$$
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hence or otherwise show that $\bar{a} \times \bar{b}, \bar{b} \times \bar{c}, \bar{c} \times \bar{a}$ are also non-coplanar.

- 11. (a) The necessary and sufficient condition for a vector point function \vec{V} having constant magnitude is that $\vec{V} \cdot \frac{d\vec{V}}{dt} = 0$.
 - (b) Find the gradient and unit harwel to the level surface $x^2 + y z = 1$ at (1,0,0)
- 12. (a) Find the divergence and curl of the vector point.

123 function
$$\vec{F} = (x^2 - y^2)\hat{i} + 2xy\hat{j} + (y^2 - xy)\hat{k}$$

(b) Prove that $div(\bar{u} \times \bar{v}) = \bar{v}.curl\,\bar{u} - \bar{u}.curl\,\bar{v}$

Group - C

- 13. (a) Find the equation of the line of action of the resultant of a system of coplanar forces acting upon a rigid load.
 - (b) A uniform beam of length 2a rests in equilibrium with one end resting against a smooth vertical wall and with a point of its length resting upon a smooth horizontal rod which is parallel to the wall and at a distance b from it. Show that inclination of the beam to

the vertical is
$$\sin^{-1}\left(\frac{b}{a}\right)^{\frac{1}{3}}$$
.

- 14. (a) Discuss the forces which can be omitted in forming the equation of the virtual work.
 - (b) A strip of length a forms the shorter diagonal of a rhomous of four uniform rods, each of length b and weight w which are hinged together. If one of the rods be supported on a horizontal position.

prove that tension of the strip is
$$\frac{2w(2b^2 - a^2)}{b\sqrt{4b^2 - a^2}}$$

- Find the tangential and normal velocity and acceleration of a particle working a plane curve.
 - (b) A particle describe the equi-angular spiral $r = ae^{in\theta}$ with a constant velocity. Find the components of the velocity and the acceleration along the radius vector and perpendicular to it.
 - 16. (a) Discuss SHM and hence find period of it.
 - (b) A particle starts with a velocity V and works under a retardation equal to K times the space described. Prove that the space traversed before

it comes to rest is equal to
$$\frac{V}{\sqrt{R}}$$
.

17. State and prove principle of conservation of linear momentum, angular momentum and energy.

(P-2000)