

2015

Time : 3 Hours

Full Marks : 75

The questions are of equal value.

Answer six questions, selecting at least three from Group-A, one from Group-B, and two from Group-C.

Group—A

1. (a) If $y = \frac{1}{x^2 + a^2}$, then find Y_n.

(b) Apply Maclaurin's series to prove

$$\sin x = x - \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} - \ldots\ldots\ldots\ldots \text{to } \infty.$$

2. (a) Evaluate : $\lim_{x \to 0}\left(\frac{\sin x}{x}\right)^{1/x^2}$

(b) Find the condition that the conics $ax^2 + by^2 = 1$, $a_1x^2 + b_1y^2 = 1$ shall cut orthogonally.

3. (a) Establish the formula $p = r\frac{dr}{dp}$ in usual symbols.

(b) Show that $\frac{x}{\log x}$ has a minimum value at $x = e$.

4. (a) Integrate by summation : $\int_a^b \sin x\,dx$.

(b) If $I_n = \int \tan^n x\,dx$, then prove that $(n-1)(I_n + I_{n-2}) = \tan^{n-1} x$.

5. (a) Trace the curve $y^2(a - x) = x^3$ and obtain the area included between the curve and its asymptotes.

(b) Find the whole length of the loop of the curve

$3ay^2 = x(x - a)^2$

6. The cardioide $r = a(1 + \cos\theta)$ revolves about the initial line. Find

(a) the volume of the solid so generated

(b) the surface area of the solid.

7. (a) Solve : $\frac{dy}{dx}=\frac{y}{x}+\tan\frac{y}{x}$

(b) Solve : $\cos^2 x\frac{dy}{dx}+y=\tan x$

8. (a) Solve : $xdx+ydy+\frac{xdy-ydx}{x^2+y^2}=0$

(b) Solve : $y = 2px + y^2p^3$

9. (a) Find the orthogonal trajectories of the family of parabolas $y^2 = 4ax$, where a is a variable parameter.

(b) Solve : $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+4y=e^x\cos x$

Group—B

10. (a) If $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$, then prove that

$$\left[\vec{a}\,\vec{b}\,\vec{c}\right]=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

(b) Prove that $\left[\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}\right]=2\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

11. (a) Prove that a necessary and sufficient condition for a vector function $\bar{a}(t)$ to have a constant direction is $\vec{a} \times \frac{d\vec{a}}{dt} = \vec{0}$

(b) If $\vec{r} = \vec{a} \cos wt + \vec{b} \sin wt$, then prove that

(i) $\vec{r} \frac{d\vec{r}}{dt} = w\,\vec{a} \times \vec{b}$ (ii) $\frac{d^2\vec{r}}{dt^2} = -w^2\vec{r}$, where $\vec{a}$ and $\vec{b}$ are constant vectors and w is also a constant.

12. (a) Prove that $\text{curl}\left(\phi\,\vec{a}\right) = \phi\,\text{curl}\,\vec{a} + (\text{grad}\,\phi) \times \vec{a}$

(b) Prove that $\text{div}\,(\text{curl}\,\vec{a}) = 0$

Group C

13. (a) Find necessary and sufficient conditions that a system of coplanar forces acting on a rigid body be in equilibrium.

(b) Forces P, Q, R act along the lines $x = 0$, $y = 0$ and $x \cos\alpha + y \sin\alpha = p$. Find the magnitude of the resultant and the equation of its line of action.

14. (a) State and prove principle of virtual work for a system of coplanar force acting at different points of a rigid body.

(b) Two equal uniform rods AB and AC, each of length 2b are freely jointed at A and rest on a smooth vertical circle of radius a. Show that, if 2θ be the angle between then, then

$$b \sin^3\theta = a \cos \theta.$$

15. (a) Establish $T^2\mu$ = constant, where the symbols have their usual meanings.

(b) A particle starts with a velocity V and moves under a retardation equal to K times the space described. Prove that the space traversed before it comes to rest is equal to $\frac{V}{\sqrt{K}}$.

16. (a) State and explain Hook's law.

(b) Find the work done in extending a light elastic string to double its length.

17. (a) Find the radial and transverse acceleration of the particle moving in a plane curve.

(b) If the radial and transverse velocities of a particle are always proportional to each other, show that the equation to the path is an equiangular spiral.

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