

2016

Time : 3 hours

Full Marks : 100

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

Answer any ***eight*** *questions,*

selecting at least ***two*** *from each group.*

Group – A

1. (a) State and prove the Taylor's theorem.

(b) If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{a}\right)^n$, prove that

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0$$

2. (a) If $u = \cos^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0$$

(b) Find the value of a and b so that

$\lim_{x\to 0}\frac{(1+ax\sin x) - b\cos x}{x^4}$ may tend to a definite limit and determine it.

3. (a) Prove that $\frac{1}{p^2} = u^2 + \left(\frac{du}{d\theta}\right)^2$.

(b) Find the radius of curvature at any point of the curve $x = a(\theta + \sin\theta)$, $y = (1 - \cos\theta)a$

4. (a) Obtain the reduction formula for $\int \sin^m x \cos^n x \, dx$

(b) Find the area of a loop of the curve $r = a\sin 3\theta$

5. (a) Find the value of $\int_a^b x^3 dx$ as the limit of the sum of a series.

(b) Evaluate $\int_0^{\pi} \frac{x \sin 2x \sin\left(\frac{\pi}{2}\cos x\right)}{2x - \pi} dx$

6. Prove that

(a) $\Gamma m \Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}} . \Gamma 2m$

(b) $\Gamma n \Gamma(1-n) = \frac{\pi}{\sin n\pi}$

7. Solve any **two** of the following :

(a) $\frac{xdx + ydy}{xdy - ydx} = \sqrt{\frac{a^2 - x^2 - y^2}{x^2 + y^2}}$

(b) $(y - 2x)dx + (6x - 5y)dy = 0$

(c) $(2x + 3y - 6)dy - (6x - 2y - 7)dx = 0$

8. Solve any **two** of the following :

(a) $x\cos\left(\frac{x}{y}\right)(ydx+xdy)=y\sin\left(\frac{y}{x}\right)(xdy-ydx)$

(b) $\left(x+2y^3\right)\frac{dy}{dx}=y$ (c) $y^2\frac{dy}{dx}=x+y^3$

9. (a) Solve the differential equation $y=p(x-b)+\frac{a}{p}$ and also find the singular solution.

(b) Find the orthogonal trajectory of the family of curves $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$

Group – B

10. (a) Show that $\left[\vec{b}\times\vec{c},\vec{c}\times\vec{a},\vec{a}\times\vec{b}\right]=\left[\vec{a}\vec{b}\vec{c}\right]$

(b) Prove that $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=(\vec{a}.\vec{c})\vec{b}-\left(\vec{a}.\vec{b}\right)\vec{c}$ and interpret geometrically the scalar triple product.

11. (a) If $\vec{a}$ and $\vec{b}$ are differentiable vector functions of a scalar t, then prove that

$$\frac{d}{dt}\left(\vec{a}.\vec{b}\right)=\vec{a}.\frac{d\vec{b}}{dt}+\frac{d\vec{a}}{dt}.\vec{b}$$

(b) Prove the necessary and sufficient condition for the vector function $\vec{a}(t)$ to have a constant magnitude is $\vec{a}.\frac{d\vec{a}}{dt}=0$.

12. (a) Prove that $\nabla \times (\vec{u} \pm \vec{v}) = \nabla \times \vec{u} \pm \nabla \times \vec{v}$

(b) $\nabla.(\phi\bar{a}) = \phi div\bar{a} + \bar{a}.grad\phi$

Group – C

13. (a) If a system of forces be equivalent to a force X along the x-axis, a force Y along y axis, and a couple G, the equation of the line of action of the resultant is $Xy - Yx + G = 0$.

(b) Show that the coditions that a particle at rest in contact with a smooth material curve is

$$X + Y\frac{dy}{dx} = 0, X\frac{dy}{dx} - Y\frac{dx}{dy} = R$$

where the symbols have their usual meanings.

14. (a) State and prove Hook's law.

(b) State and prove the principle of conservation of linear momentum.

15. (a) Find the forces which may be omitted in forming the equation of virtual work.

(b) The middle points of the opposite sides of a jointed quadrilateral are connected by light rods of lengths l and l'. If T and T' be the tension in these rods, prove that $\frac{T}{l} + \frac{T'}{l'} = 0$.

16. (a) Show that the sum of kinetic energy and potential energy is constant throughout the motion, when a particle moves under the action of a conservative system of forces.

(b) Define intertial frames of reference. Also state the two laws of mechanics.

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