

2017

Time : 3 hours

Full Marks : 100

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

Answer any ***eight*** *questions,*

selecting at least ***two*** *from each group.*

Group – A

1. (a) State and prove Euler's theorem for function of two variables.

   (b) Find the radius of curvature for the pedal curve $P = f(r)$.

2. (a) If $y = \cos(ax + b)$; find y_n.

   (b) If $y = \dfrac{x}{x^2 + a^2}$; find y_n.

3. (a) Evaluate $\underset{x \to 0}{Lt} \left(\dfrac{\tan x}{x} \right)^{\frac{1}{x^2}}$

(b) If $y = \sin\left(m\sin^{-1} x\right)$, prove that

$$\left(1-x^2\right)y_2 - xy_1 + m^2 y = 0$$

4. Show that $\int_0^1 \frac{\log(1+x)}{1+x^2}\,dx = \frac{\pi}{8}\log 2$

5. (a) Evaluate $\int_0^{\frac{\pi}{2}} \log\sin x\,dx$

(b) Prove that $\int_0^{\infty} \frac{\sin mx}{x}\,dx = \frac{\pi}{2}$

6. Show that the entire area of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ is $\frac{3}{8}\pi a^2$.

7. (a) Solve : $x\left(y^2+1\right)dx + y\left(x^2+1\right)dy = 0$

(b) Solve : $\frac{dy}{dx} + xy = x^3$

8. (a) Solve : $\frac{dy}{dx} - y\cos x = 2\sin 2x$

(b) Solve : $\frac{dy}{dx} = \frac{3x+2y}{2x-3y}$

9. (a) Solve : $y + px = x^4 p^2$

(b) Solve : $(y-1)p - xp^2 + 2 = 0$

Group – B

10. (a) Define scalar triple product of three vectors and cross product of three vectors.

(b) Prove that $[a+b \quad b+c \quad c+a] = 2[a\,b\,c]$

11. For any three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ prove that

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$$

12. (a) If $\vec{r} = \vec{a}\cos\omega t + \vec{b}\sin\omega t$, prove that

$$\frac{d^2\vec{r}}{dt^2} = -\omega^2\vec{r}\,.$$

(b) Prove that $\nabla.(r^n\vec{r}) = (n+3)r^n$

Group – C

13. Define astatic centre of a system of coplanar forces and obtain its position.

14. (a) Two system of forces P,Q,R and P', Q', R' act along the sides BC, CA and AB of a triangle ABC, prove that their resultant will be parallel

if $\begin{vmatrix} \sin A & \sin B & \sin C \\ P & Q & R \\ P' & Q' & R' \end{vmatrix} = 0$

(b) State and prove the principle of virtual work for any system of forces in one plane.

15 (a) Define simple Harmonic motion. Find its periodic time.

(b) Prove that radial and transverse velocities along the path are $\frac{dr}{dt}$ and $r\frac{dv}{dt}$.

16. (a) Find the work done in extending a light elastic string to double its length.

(b) Show that the central orbit is a plane curve.

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