2018

Full Marks : 100

Time : 3 hours

The questions are of equal value

Answer **eight** questions, selecting at least **two** from each Group

Group—A

1. (a) If

$$y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$$

then prove that

$$(x^2 - 1) y_{n+2} + (2n+1) x y_{n+1} - (n^2 - m^2) y_n = 0$$

(b) If

$$u = \tan^{-1} \frac{x^2 + y^2}{x - y}, \ x \neq y$$

then show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$$

2. *(a)* Evaluate :

$$\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right)$$

(b) Find the equation of the tangent at (x, y) to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

3. *(a)* Find the Cartesian formula for radius of curvature.

(b) Find those asymptotes of the following curve which are parallel to the coordinate axes $x^2y^2 = a^2(x^2 + y^2)$.

4. *(a)* Prove that

$$\lim_{n \to \infty} \left[\frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{3n} \right] = \log 3$$

(b) Integrate

$$\int \sin^5 x \, dx$$

5. *(a)* Prove that

$$\int_0^{\pi/2} \sin^4 x \cos^2 x \, dx = \frac{\pi}{32}$$

(b) Evaluate

$$\int_a^b (x-a)^m (b-x)^n \, dx$$

m and n being positive integer.

6. Find the surface of the solid generated by the revaluation of the curve $x = a\cos^3 t$ and $y = a\sin^3 t$ about the x-axis.

7. *(a)* Solve :

$$\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$$

(b) Solve :

$$x(x - y) \, dy + y^2 dx = 0$$

8. *(a)* Solve :

$$(1 + y^2) dx = (\tan^{-1} y - x) \, dy$$

(b) Solve :

$$y = px + ap(1 - p)$$

9. *(a)* Find the orthogonal trajectory of $r = a(1 + \cos\theta)$.

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(b) Solve the equation :

$$(D^3 - D^2 - 4D + 4)y = e^{3x}$$

Group—B

10. *(a)* Prove that

$$\bar{a} \times (\bar{b} \times \bar{c}) + \bar{b} \times (\bar{c} \times \bar{a}) + \bar{c} \times (\bar{a} \times \bar{b}) = 0$$

(b) If

$$\bar{r} = \bar{a}e^{nt} + \bar{b}e^{-nt}$$

where $\bar{a}$ and $\bar{b}$ are constant vectors, then show that

$$\frac{d^2\bar{r}}{dt^2} - n^2\bar{r} = 0$$

11. *(a)* If $\frac{d\bar{a}}{dt} = \bar{r} \times \bar{a}$ and $\frac{d\bar{b}}{dt} = \bar{r} \times \bar{b}$, then show that

$$\frac{d}{dt}(\bar{a} \times \bar{b}) = \bar{r} \times (\bar{a} \times \bar{b})$$

(b) If $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then prove that $\nabla r^n = nr^{n-2}\bar{r}$.

12. *(a)* Prove that

$$\operatorname{div}(\bar{a} \times \bar{b}) = \bar{b} \cdot \operatorname{curl} \bar{a} - \bar{a} \cdot \operatorname{curl} \bar{b}$$

(b) Prove that

$$\text{curl grad}\,\phi = 0$$

Group—C

13. State and prove the principle of virtual work for a system of coplanar forces acting at different points of a rigid body.

14. Show that a system of coplanar forces acting in one plane at different points of a rigid body can be reduced to a single force through only given point and a single couple.

15. *(a)* Show that if a particle is acted on by a conservative system of force and be in motion, then the sum of the kinetic and potential energies of the particle remains constant.

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(b) A particle move in a straight line with a constant acceleration f, starting from same point in the line with a velocity u, to discuss the motion.

16. (a) A point in a straight line with SHM has velocities v_1 and v_2 when its distances from the centre are x_1 and x_2. Show that the period of motion is

$$2\pi\sqrt{\frac{x_1^2 - x_2^2}{v_2^2 - v_1^2}}$$

(b) A particle is suspended from a fixed point by an elastic string; to discuss the motion.

17. (a) The velocities of a particle along and perpendicular to the radius from a fixed origin are λr and $\mu\theta$, find the path and show that the acceleration along and perpendicular to the radius vector, are

$$\lambda^2 r - \frac{\mu^2\theta^2}{r} \text{ and } \mu\theta\left(\lambda + \frac{\mu}{r}\right)$$

(Continued)

(b) A particle moves in a plane curve, so that its tangential and normal accelerations are equal and the angular velocity of the tangent is constant. Find the curve.

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