1404-02 **Total Pages : 8**

Degree (Part-II) Examination, 2022

(Vocational - Subsidiary)

MATHEMATICS

[PPU-D-II-(SUB)-MATH]

Time : Three Hours] **[Maximum Marks : 100**

Note : Candidates are required to give their answers in their own words as far as practicable. The questions are of **equal** value. Answer **any five** questions in all. Question no. 1 is **compulsory**. Besides this, attempt one question from each Group.

1. Choose the correct option of the following :

(i) If $y = \sqrt{\sin x}$ then $\frac{dy}{dx}$ is :

(a) $\frac{1}{2}\sin x.\cos x$

(b) $\sqrt{\cos x}$

(c) $\sqrt{\sin x}.\cos x$

(d) None of these

(ii) If $y = x\sin(a+y)$ then $\frac{dy}{dx}$ is

(a) $\frac{\sin^2(a+y)}{\sin a}$

(b) $\frac{\sin a}{\sin^2(a+y)}$

(c) $\frac{\sin(a+y)}{\cos a}$

(d) None of the above

(iii) If $y = \sin(\log x)$ then $\frac{dy}{dx}$ is :

(a) $\frac{\log x}{\sin x}$

(b) $\frac{1}{x}\cdot\cos(\log x)$

(c) $\cos(\log x)$

(d) None of these

(iv) If $x^x = y^y$ then $\frac{dy}{dx}$ is :

(a) $-\frac{y}{x}$

(b) $-\frac{x}{y}$

(c) $\frac{1+\log x}{1+\log y}$

(d) None of the above

(v) $\int \log x \, dx$ is equal to :

(a) $x \log x - x + c$

(b) $\frac{1}{x} + c$

(c) $x \log x + c$

(d) None of the above

(vi) If $x\frac{dy}{dx}+3y=x$ then solution is :

(a) $x^3y=\frac{x^4}{4}+c$

(b) $x^2y^2=c$

(c) $xy+y^2+c=0$

(d) None of the above

(vii) If $\frac{dy}{dx}+y\log x=x^2$ then solution is :

(a) $y=\frac{x^3}{4}+c$

(b) $y=\frac{x^4}{4}+c$

(c) $y=x+c$

(d) None of the above

(viii) The equation of line of action of the resultant of coplanar forces is :

(a) $Yx - Xy = 0$

(b) $Yx - Xy + G = 0$

(c) $Xx - Yy + G = 0$

(d) None of the above

(ix) Change of velocity with respect to time is :

(a) Force

(b) Displacement

(c) Acceleration

(d) None of the above

(x) The differential equation of S.H.M. is :

(a) $\frac{d^2x}{dt^2} = -\mu x$

(b) $\frac{dx}{dt} = -\mu x$

(c) $\frac{d^2x}{dt^2} = \mu x$

(d) None of these

Group-A

2. (a) State and prove Leibnitz theorem on successive differentiation.

(b) If $y = \sin(ax + b)$ then find y_n.

3. (a) Evaluate $\int \frac{x \sin^{-1} x}{\sqrt{1 - x^2}} dx$.

(b) Show that $\int_0^1 \frac{\log(1+x)}{1+x^2} dx = \frac{\pi}{8} \log 2$.

4. (a) Solve $\frac{dy}{dx}(1 + x^2) \cdot \tan^{-1} x + y = 0$.

(b) Solve $\frac{dy}{dx} = \frac{y}{x} + \tan \frac{y}{x}$.

5. (a) Solve $x \frac{dy}{dx} + 3y = x$.

(b) Solve $(x+1)\cdot\frac{dy}{dx}+1=2e^{-y}$.

Group-B

6. (a) Write down the necessary and sufficient condition that the three non-parallel, non-zero vectors $\vec{a},\vec{b},\vec{c}$ be coplanar.

(b) Prove that $(\vec{a}\times\vec{b})\times\vec{c}=(\vec{a}\cdot\vec{c})b-(\vec{b}\cdot\vec{c})\vec{a}$.

If $\vec{a},\vec{b},\vec{c}$ are three non-coplanar vectors then prove that $\vec{b}\times\vec{c},\vec{c}\times\vec{a},\vec{a}\times\vec{b}$ are also non-coplanar.

Group-C

8. (a) Find the equation of the line of action of the resultant of coplanar system of forces acting on a rigid body.

(b) Obtain necessary and sufficient conditions for the equilibrium of a system of coplanar forces acting on a rigid body.

9. (a) Find the radial velocity of a particle P(r, θ) describing a smooth curve.

(b) Find the radial acceleration of a particle $P(r, \theta)$ describing a smooth curve.

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