

2017

Time : 3 hours

Full Marks : 100

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

Answer any **eight** questions, selecting at least **one** from each group.

Group – A

1. For any three sets A, B and C, Prove any two of the following :-

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(iii) $A \times (B - C) = (A \times B) - (A \times C)$

2. (a) Define equivalence relation with example.

(b) Define one-one on to mapping with example.

3. (a) Define abelian group and prove that set of even integers is an abelian additive group.

(b) State and Prove Cayley's theorem.

4. (a) Prove that every group of prime order is cyclic.

(b) Define Ring, Integral Domain and Field.

Group – B

5. (a) Prove that product of two matrices is not commutative.

(b) Find the inverse of the matrix A

$$\text{Where } A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 6 & 7 & 9 \end{bmatrix}$$

6. (a) Find the rank of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$$

(b) Define Hermitian and skew Hermitian matrix.

7. (a) Prove that the sphere is a convex set.

(b) Prove that the intersection of two convex sets is also a convex set.

8. Solve the following L.P.P graphically

Max $Z = 5x_1 + 7x_2$ subject to the constraints

$$x_1 + x_2 \leq 4$$

$$3x_1 + 8x_2 \leq 24$$

$$10x_1 + 7x_2 \leq 35 ; x_1 \geq 0 ; x_2 \geq 0$$

Group – C

9. State and Prove De' Moivre's theorem .

10. (a) State and Prove Gregories series.

(b) Find the expansion of $\cosh \theta$.

11. (a) State and prove Cauchy's general principle of convergence.

(b) State and Prove Cauchy's root test.

12. State and Prove De' Alembert's ratio test regarding convergence of a series.

Group – D

13. (a) Define limiting point circle and find its limiting points.

(b) Find the conditions that two circles touch each other.

14. (a) Define co-axial system of circles and find its equation.

(b) Find the standard equation of parabola.

Group – E

15. (a) Define Direction cosines of a line. If l, m, n be the direction cosines of a line, Prove that $l^2 + m^2 + n^2 = 1$.

(b) Find the angle between two st. lines whose direction cosines are l_1, m_1, n_1 and l_2, m_2, n_2 .

16. (a) Find the equation of plane in intercept form.

(b) Find the equation of st. line in symmetrical form.

Or

Prove that equation of St. line in symmetrical

$$\text{form is } \frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$$

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