2009

Full Marks : 100

Time : 3 hours

The questions are of equal value

Answer **eight** questions, selecting at least **two** from each Group

Group—A

1. (a) State and prove Maclaurin's theorem.

(b) If $y = e^{a \sin^{-1} x}$, prove that

$$(1 - x^2) y_{n+2} - (2n+1) x y_{n+1} - (n^2 + a^2) y_n = 0$$

2. (a) Evaluate :

$$\lim_{x \to 0} \left(\frac{1}{x} - \cot x \right)$$

(b) If $u = \dfrac{x - y}{x + y}$, prove that $x \dfrac{\partial u}{\partial x} + y \dfrac{\partial u}{\partial y} = 0$.

3. (a) Prove that

$$\rho = r \frac{dr}{dp}$$

where the symbols have their usual meanings.

(b) Prove that the sum of the intercepts of the tangents to $\sqrt{x} + \sqrt{y} = \sqrt{a}$ upon the coordinate axes is constant.

4. (a) Evaluate

$$\int_a^b \cos x \, dx$$

as a limit of a sum.

(b) Evaluate :

$$\int_0^{\pi/2} \log \sin x \, dx$$

5. (a) Obtain the reduction formula for $\int \tan^n x \, dx$.

(b) Find the entire length of the cardioid $r = a(1 + \cos\theta)$.

6. Define Beta and Gamma functions. Prove that

$$\frac{\Gamma(n)\Gamma(m)}{\Gamma(m+n)} = 2\int_0^{\pi/2} (\sin\theta)^{2m-1} (\cos\theta)^{2n-1} \, d\theta$$

7. Solve *any two* of the following :

(i) $\dfrac{dy}{dx} = \dfrac{(1+x)\,y^2}{x^2(1+y)}$

(ii) $x\frac{dy}{dx} = y + x\tan\frac{y}{x}$

(iii) $\frac{dy}{dx} + 2xy = 2x^3$

8. Solve *any two* of the following :

(i) $(x - y - 2)\,dx = (2x - 2y - 3)\,dy$

(ii) $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$

(iii) $(3x - 2y)\,dy = (2x - 3y)\,dx$

9. (a) Solve $y = 2px + 4xp^2$ and obtain singular solution, where $p = \frac{dy}{dx}$.

(b) Find the orthogonal trajectories of the family of parabolas $y^2 = 4ax$.

Group—B

10. (a) Prove that

$$[\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}] = 2[\vec{a}\ \vec{b}\ \vec{c}]$$

(b) Find the volume of a parallelopiped whose concurrent edges are expressed by the vectors $\vec{i}+2\vec{j}+3\vec{k}$, $3\vec{i}+7\vec{j}-4\vec{k}$ and $\vec{i}-5\vec{j}+3\vec{k}$.