

2010

Time : 3 hours

Full Marks : 100

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

Answer eight questions, selecting at least two from each Group.

Group – A

1. (a) State and prove Taylor's theorem.

   (b) If $y = (\sin^{-1} x)^2$, Prove that $(1 - x^2)\, y_{n+2} - (2x + 1)\, xy_{n+1} - n^2 y_n = 0$

2. (a) Evaluate :

$$\lim_{x \to 0} \left(\frac{\tan}{x}\right)^{\frac{1}{x^2}}$$

(b) If $u = \tan^{-1}\frac{x^3 + y^3}{x - y}$ prove that $\frac{x.\delta u}{\delta x} + \frac{y.\delta u}{\delta y}$ $= \sin 2u$.

3 (a) Find the equation of the tangent at (a, b)

$$\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$$

(b) Show that in the curve $y = be^{-ax}$ the sub tangant varies at the square of the abscissa

4 (a) Evaluate :

$$\int_{\alpha}^{\beta} \frac{dx}{\sqrt{(x-\alpha)(x-\beta)}}$$

(b) Evaluate :

$$\int_{\alpha}^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} . dx$$

5. (a) Obtain the reduction formula for $\int_0^{\pi/4} \cot^n x$.

dx, where n = 1

(b) Find the length of the Parabola $\frac{2a}{r} = 1 + \cos\theta$ cut off by the latu rectum.

(b) A particle starts from the origin and components of its velocity parallel to x and z-axis at time t, are 2t + 3 and 4t respectively Find the path.

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(b) Six equal rods AB, BC, CD, EF and FA are each of weight W and freely jointed at their extremities so as the form a hexagon, the rod AB is fixed in a horizontal position and the middle point of AB and DE are joined by string. Prove the its tension

15 (a) A particle starts with a velocity V and moves under a retardation equal to K-times the space described. Prove that the space traversed before it comes to rest is equal to

(b) Find the work done in extending a light elastic string to double its length.

16 (a) A particle is moving in a plane curve. Find the components of velocity at time t along and perpendicular to the radius vector drawn from a fined point in the plane.

6. (a) Prove the following :

$B(m, n) = B(n, m)$.

(b) Prove the following :

$$\int_0^{\infty} \frac{x^{m-1}}{(1+n)^{m+n}}\, dx = B(m, n).$$

7. Solve any two of the following :

(a) $x^2y\, dx - (x^2 + y^2)\, dy = 0$

(b) $\frac{dy}{dx} = \frac{6x - 2y - 7}{3x - y + 4}$

(c) $\frac{dy}{dx} = \frac{y(x - 2y)}{x(x - 3y)}$

8. Solve any two of the following :

(a) $(2x + 3y - 5)\frac{dy}{dx} + 2x + 3y - 1 = 0$

(b) $2xy\frac{dy}{dx} = x^2 + y^2$

(c) $x\cdot\frac{dy}{dx} + 4y = x^6$