 XEV (S-2) – M (2)

2014

Time : 3 hours

Full Marks : 100

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

*Answer any **eight** questions, selecting at least **two** from each Group.*

Group – A

1. (a) State and prove Maclaurin's theorem.

   (b) If $Y = e^{a\sin^{-1}x}$, Prove that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$$

2. (a) Evaluate :

$$\lim_{x\to 0}\left(\frac{1}{x} - \cot x\right)$$

   (b) If $u = \frac{x-y}{x+y}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

3. (a) Prove that

$$\rho = r\frac{dr}{dp}$$

where the symbols have their usual meanings.

(b) Prove that the sum of the intercepts of the tangents to $\sqrt{x} + \sqrt{y} = \sqrt{a}$ upon the co-ordinate axes is constant.

4. (a) Evaluate

$$\int_a^b \cos x \, dx$$

as a limit of a sum.

(b) Evaluate :

$$\int_0^{\pi/2} \log \sin x \, dx$$

5. (a) Obtain the reduction formula for

$$\int \tan^n x \, dx$$

(b) Find the entire length of the cardioid $r = a(1 + \cos\theta)$.

6. Define Beta and Gamma functions. Prove that

$$\frac{\Gamma(n)\Gamma(m)}{\Gamma(m+n)} = 2\int_0^{\pi/2} (\sin\theta)^{2m-1}(\cos\theta)^{2n-1} d\theta$$

7. Solve any **two** of the following :

(i) $\frac{dy}{dx} = \frac{(1+x)y^2}{x^2(1+y)}$

(ii) $x\frac{dy}{dx} = y + x\tan\frac{y}{x}$

(iii) $\frac{dy}{dx} + 2xy = 2x^3$

8. Solve any **two** of the following :

(i) $(x-y-2)dx = (2x-2y-3)dy$

(ii) $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$

(iii) $(3x-2y)dy = (2x-3y)dx$

9. (a) solve $y = 2px + 4xp^2$ and obtain singular solution, where $p = \frac{dy}{dx}$.

(b) Find the orthogonal trajectories of the family of parabolas $y^2 = 4ax$.

Group – B

10. (a) Prove that

$$\left[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}\right] = 2\left[\vec{a}\,\vec{b}\,\vec{c}\right]$$

(b) Find the volume of a paralelopiped whose concurrent edges are expressed by the vectors $\bar{i}+2\bar{j}+3\bar{k}, 3\bar{i}+7\bar{j}-4\bar{k}$ and $\bar{i}-5\bar{j}+3\vec{k}$.

11. (a) If $\bar{a}$ and $\bar{b}$ are differentiable vector functions of a scalar t, then show that

$$\frac{d}{dt}(\bar{a}\times\bar{b})=\bar{a}\times\frac{d\bar{b}}{dt}+\frac{d\bar{a}}{dt}\times\bar{b}$$

(b) Prove that the necessary and sufficient condition for the vector function $\bar{a}(t)$ to have constant direction is $\bar{a}\times\frac{d\bar{a}}{dt}=0$.

12. (a) Prove that

$$\text{grad }(\phi\psi)=\phi\text{ grad }\psi+\psi\text{ grad }\phi$$

(b) Prove that

$$\text{curl (grad) }\phi=0$$

Group – C

13. (a) Obtain the equation to the line of action of the resultant of a system of coplanar forces.

(b) Three forces P, Q, R act along the sides of a triangle formed by the lines $x + y = 1$; $y - x = 1$ and $y = 2$. Find the equation to line of action of the resultant.

14. (a) State and prove the converse of the principle of virtual work for any system of the force in one plane.

(b) Three forces P, Q, R act along the sides BC, AC, BA of an equilateral triangle ABC. If their resultant is a force parallel to BC through centroid of the triangle, then prove that $\frac{1}{2} P = Q = R$.

15. (a) State and prove Hooke's law.

(b) State and prove the principle of conservation of linear momentum.

16. (a) When a particle moves under the action of a conservative system of forces the sum of KE and PE is constant throughout the motion. Prove it.

(b) If a particle is moving in a plane curve, then find component of its acceleration along the tangent and the normal to the curve at any instant.

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