

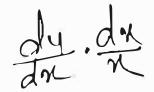
Full Marks: 100

Time: 3 hours

The questions are of equal value

Answer eight questions, selecting at least two from each Group

Group—A



1. (a) If

$$y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$$

then prove that

$$(x^2-1)y_{n+2}+(2n+1)xy_{n+1}-(n^2-m^2)y_n=0$$

$$u = \tan^{-1} \frac{x^2 + y^2}{x - y}, \ x \neq y$$
 then show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$$

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(Turn Over)

2. (a) Evaluate:

$$\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right)$$

(b) Find the equation of the tangent at (x, y) to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

- 3. (a) Find the Cartesian formula for radius of curvature.
 - (b) Find those asymptotes of the following curve which are parallel to the coordinate axes $x^2y^2 = a^2(x^2 + y^2)$.

4. (a) Prove that

$$\lim_{n \to \infty} \left[\frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{3n} \right] = \log 3$$

(b) Integrate

$$\int \sin^5 x \, dx$$

5. (a) Prove that

$$\int_0^{\pi/2} \sin^4 x \cos^2 x \, dx = \frac{\pi}{32}$$

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(Continued)

(b) Evaluate
$$\int_a^b (x-a)^m (b-x)^n dx$$

m and n being positive integer.

6. Find the surface of the solid generated by the revaluation of the curve $x = a \cos^3 t$ and $y = a \sin^3 t$ about the x-axis.

7. (a) Solve:

$$\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$$

(b) Solve:

$$x(x-y)\,dy+y^2dx=0$$

8. (a) Solve:

$$(1+y^2)dx = (\tan^{-1} y - x) dy$$

(b) Solve:

$$y = px + ap (1 - p)$$

9. (a) Find the orthogonal trajectory of $r = \alpha (1 + \cos \theta)$.

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(b) Solve the equation :

$$(D^3 - D^2 - 4D + 4)y = c^{3x}$$

Group-B

10. (a) Prove that

$$\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = 0$$

(b) If

$$\bar{r} = \bar{a}e^{nt} + \bar{b}e^{-nt}$$

where \bar{a} and \bar{b} are constant vectors, then show that

$$\frac{d^2\bar{r}}{dt^2} - n^2\bar{r} = 0$$

11. (a) If $\frac{d\overline{a}}{dt} = \overline{r} \times \overline{a}$ and $\frac{d\overline{b}}{dt} = \overline{t} \times \overline{b}$, then show that

$$\frac{d}{dt}(\overline{a}\times\overline{b})=\overline{r}\times(\overline{a}\times\overline{b})$$

(b) If $\vec{r} = x \hat{i} + y \hat{j} + z \hat{k}$, then prove that $\nabla r^n = n r^{n-2} \tilde{r}$.

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(Continued)

12. (a) Prove that

$$\operatorname{div}(\tilde{a} \times \tilde{b}) = \tilde{b} \cdot \operatorname{curl} \tilde{a} - \tilde{a} \cdot \operatorname{curl} \tilde{b}$$

(b) Prove that

- 13. State and prove the principle of virtual work for a system of coplanar forces acting at different points of a rigid body.
- 14. Show that a system of coplanar forces acting in one plane at different points of a rigid body can be reduced to a single force through only given point and a single couple.
- 15. (a) Show that if a particle is acted on by a conservative system of force and be in motion, then the sum of the kinetic and potential energies of the particle remains constant.

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(Turn Over)

- (b) A particle move in a straight line with a constant acceleration f_i starting from same point in the line with a velocity u_i to discuss the motion.
- 16. (a) A point in a straight line with SHM has velocities v_1 and v_2 when its distances from the centre are x_1 and x_2 . Show that the period of motion is

$$2\pi \sqrt{\frac{x_1^2 - x_2^2}{v_2^2 - v_1^2}}$$

- (b) A particle is suspended from a fixed point by an elastic string; to discuss the motion.
- 17. (a) The velocities of a particle along and perpendicular to the radius from a fixed origin are λr and $\mu\theta$, find the path and show that the acceleration along and perpendicular to the radius vector, are

$$\lambda^2 r - \frac{\mu^2 \theta^2}{r}$$
 and $\mu \theta \left(\lambda + \frac{\mu}{r} \right)$

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(Continued)

(h) A particle moves in a plane curve, so that its tangential and normal accelerations are equal and the angular velocity of the tangent is constant. Find the curve.

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XEV (S-2) - Math